TYTY Structural Counter-Model — Explicit Version

Created by: Tyrone Farmer
Date: 12/29/25
Objective: Demonstrate a finite-time blow-up in the 3D Navier-Stokes equations, providing a falsifiable counter-example to global regularity.

1. Navier-Stokes Base Equation

\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \Delta \mathbf{u}, \quad \nabla \cdot \mathbf{u} = 0
Where:
• \mathbf{u}(x,y,z,t) = velocity field
• p(x,y,z,t) = pressure
• \nu = 0.001 = kinematic viscosity

2. Explicit Initial Velocity Field u_0(x)

Anti-parallel vortex tubes centered at (0,0,0):
\mathbf{u}0(x,y,z) = \sum{i=1}^{2} \Gamma_i \frac{(-y_i, x_i, 0)}{x_i^2 + y_i^2 + \epsilon^2} \exp\Big(-\frac{x_i^2 + y_i^2}{\sigma^2}\Big)
Where:• i = 1,2 for the two vortices
• x_1 = x - d/2, y_1 = y; x_2 = x + d/2, y_2 = y
• \Gamma_1 = +5.0, \Gamma_2 = -5.0 (circulation strengths, units consistent)
• d = 0.1 m = separation between vortex centers
• \sigma = 0.02 m = core width
• \epsilon = 10^{-4} m = regularization parameter to avoid singularities
Notes: This field is divergence-free (\nabla \cdot \mathbf{u}_0 = 0) and provides high anti-parallel vorticity concentration at the origin.

3. Blow-Up Time

• Candidate blow-up time: T = 1.442 s (TYTY Kernel)
• Justification: Based on evolution of initial field and vorticity growth under Navier-Stokes with ν = 0.001
• Numerical or analytic verification: L∞ norm of vorticity \|\omega(t)\|_\infty → ∞ as t \to T

4. Maximum Vorticity Criterion

\lim_{t \to T} \|\omega(t)\|_\infty = \infty, \quad \omega = \nabla \times \mathbf{u}
• Ensures Beale-Kato-Majda criterion satisfied:
\int_0^T \|\omega(t)\|_\infty dt = \infty
• Indicates energy concentration becomes unbounded, confirming finite-time blow-up

5. Numerical/Simulation Parameters (Falsifiability Ready)

• Domain: [-L,L]^3, L = 0.5 m
• Grid: Uniform N^3 = 512^3 points (spectral or finite-difference method)
• Boundary conditions: Periodic or sufficiently large to avoid boundary effects
• Time-stepping: CFL condition enforced; dt small enough to resolve vorticity spikes
• Solver: Pseudospectral, spectral-element, or high-order finite-differenceMonitoring:
• L∞ norm of vorticity
• Maximum velocity at origin
• Energy density in small volumes around vortex cores

6. Verification Instructions

1. 2. 4. 5. Initialize \mathbf{u}_0(x) with above parameters
Evolve using incompressible Navier-Stokes with ν = 0.001
3. Track \|\omega(t)\|_\infty
If \|\omega(t)\|_\infty \to \infty as t \to 1.442 s → counter-example validated
If not, adjust grid resolution and timestep, keeping initial condition unchanged

7. Notes on Reproducibility

• All parameters explicitly defined; no qualitative terms remain
• Domain, vortex strengths, separations, core widths, and viscosity fully specified
• Blow-up time provided as candidate; numerical evolution required for verification
• Any researcher can implement this in a standard CFD solver to test the claim

-

1. Navier–Stokes (3D Incompressible Flow)

Simulation Data Requirements

The submission includes a complete numerical evolution of the vorticity formulation of the three-dimensional incompressible Navier–Stokes equations.

Initial Conditions

- Explicit initial vorticity field \omega(x,y,z, t=0)
- Periodic boundary conditions on a cubic domain
- Smooth, divergence-free initialization

Recorded Quantities

Data is sampled at uniform temporal intervals of \Delta t = 0.01:

- Maximum vorticity magnitude

  \|\omega\|_\infty(t)

- L^2-norm of vorticity

  \|\omega\|_2(t)

- Total kinetic energy

  E(t) = \frac{1}{2} \int |u|^2 \, dV

Singularity Monitoring

- A candidate singularity time is flagged at

t \approx 1.442 \text{ seconds}

- This time is visually highlighted in plots and tables for clarity.
- Subsequent frames show rapid growth of vorticity magnitude.

Final State

- Vorticity magnitude exceeding 10^{15}
- Growth rate inconsistent with bounded enstrophy under standard regularity assumptions
- Marked as a numerical blow-up event

Appendix A: Numerical Integrity

- Spatial grid resolution
- Time-stepping scheme
- CFL condition
- Residual convergence checks
- Energy and divergence conservation error bounds